Velocity-selective coherent population trapping

Velocity-selective coherent population trapping (VSCPT) \((\text{Aspect et al. 1988})\) is a mechanism of laser cooling below the one photon recoil limit. It is based on coherent trapping of atomic populations. Let us consider a three-level \(\Lambda\)-shape configuration with two degenerated ground Zeeman sublevels \(g_{\pm} (m=\pm)\) coupled to the excited level \(e_0 (m=0)\) by two counter-propagating \(\sigma_+\) and \(\sigma_-\) circularly polarized laser beams with the same frequency \(\omega_L\) and the same intensity (Fig. 1).

![Figure 1. Three level \(\Lambda\) configuration. The Clebsch-Gordan coefficients are equal to \(\pm \sqrt{2}\).

If an atom is at rest, two-photon Raman processes result to a nonabsorbing coherent superposition of \(g_+\) and \(g_-\) sublevels. If the atom moves along \(z\)-axis, opposite Doppler shifts on the two counter-propagating laser beams break the Raman resonance condition. Let us consider this process in details. The state \(|e_0, p\rangle\) describes an atom in the level \(e_0\) with the value \(p\) of the \(z\)-component, \(P_z\), of the atomic momentum \(\mathbf{p}\).

The state \(|e_0, p\rangle\) is coupled to the states \(|g_-, p-h\mathbf{k}\rangle\) and \(|g_+, p+h\mathbf{k}\rangle\) by stimulated emission of a \(\sigma_+\) and \(\sigma_-\) laser photon carrying a momentum \(-h\mathbf{k}\) and \(+h\mathbf{k}\), respectively, if spontaneous emission is not taken into account. Thus, for each value of \(p\) there is a family of three states, \(|e_0, p\rangle\), \(|g_-, p-h\mathbf{k}\rangle\) and \(|g_+, p+h\mathbf{k}\rangle\), which are coupled by the interaction Hamiltonian \(V\)

\[
\langle g_\pm, p \pm h\mathbf{k}| V |e_0, p\rangle = \mp \hbar \Omega_{Rabi}/2 \exp(i\omega_L t),
\]

where \(\omega_{Rabi}\) is the Rabi frequency. \(V = -\mathbf{D} \cdot \mathbf{E}(z,t)\), here \(\mathbf{D}\) is the electric-dipole-moment operator and

\[
E(z,t) = \frac{1}{2} \{ e_+ \mathbf{E}_+ [\exp i(kz - \omega_L t) + \text{c.c.}] \} + \frac{1}{2} \{ e_- \mathbf{E}_- [\exp i(kz - \omega_L t) + \text{c.c.}] \},
\]

is the classical electric field, where \(e_+ = (e_+ + i e_-)/\sqrt{2}\) corresponds to a \(\sigma_+\) circularly polarized wave propagating towards \(z>0\), and corresponds \(e_- = (e_+ - i e_-)/\sqrt{2}\) to a \(\sigma_-\) circularly polarized wave propagating towards \(z<0\). If \(p \neq 0\), the difference
between the kinetic energy, \((p + \hbar k)/2M\), of the sate \(|g_-, p + \hbar k\rangle\) and the kinetic energy, \((p - \hbar k)/2M\), of the sate \(|g_-, p - \hbar k\rangle\), is \(2\mu \hbar k/M\). The nonabsorbing trapping state can be presented as

\[
|\psi_{NA}(0)\rangle = (|g_-, -\hbar k\rangle + |g_+, +\hbar k\rangle)/\sqrt{2}.
\] (3)

It is stationary. Indeed, the states \(|g_-, -\hbar k\rangle\) and \(|g_+, +\hbar k\rangle\) have the same internal and kinetic energies, and \(\langle \psi_{NA}(0)|v|e_0, 0\rangle = 0\). It is important to emphasise that an atom pumped in the state \(|\psi_{NA}(0)\rangle\) remains trapped there indefinitely (coherent population trapping) and atomic momentum distribution has two peaks at \(\pm \hbar k\). If \(p \neq 0\), the nonabsorbing and absorbing states can be presented as

\[
|\psi_{NA}(p)\rangle = (|g_-, p - \hbar k\rangle + |g_+, p + \hbar k\rangle)/\sqrt{2},
\]

\[
|\psi_{A}(p)\rangle = (|g_-, p - \hbar k\rangle - |g_+, p + \hbar k\rangle)/\sqrt{2},
\] (4a)

respectively. The state \(|\psi_{NA}(p)\rangle\) is not a trapping state. It is not stationary. Indeed, the energy difference between the states \(|g_-, p - \hbar k\rangle\) and \(|g_+, p + \hbar k\rangle\) is \(2\mu \hbar k/M\). An atom oscillate between the states \(|\psi_{NA}(p)\rangle\) and \(|\psi_{A}(p)\rangle\) at the frequency \(2\mu k/M\). The absorption rate from the state \(|\psi_{NA}(p)\rangle\) is of the order of \(\gamma'' = \gamma(kp/\omega_{Rabi}M)\) for \(\omega_{Rabi} \ll \gamma\), where \(\gamma\) is the spontaneous emission rate from the excited state. It means as soon as \(p\) decreases the absorption rate, \(\gamma''\), from the state \(|\psi_{NA}(p)\rangle\) decreases (velocity-selective coherent population trapping). Spontaneous emission can actually redistribute atoms between different \(p\)-families. The one photon recoil momentum along \(z\)-axis is a random variable between \(-\hbar k\) and \(+\hbar k\). This mechanism provides the pumping and accumulation of atoms into the non absorbing state \(|\psi_{NA}(p)\rangle\) with very small \(p\) or \(p=0\). It is important to emphasise that the recoil of the spontaneous-emission photon is part of the cooling process. It means the one-photon recoil energy is not a limit in this case. The final temperature in this scheme is limited only by the coherent interaction time. By increasing the coherent interaction time, still narrower velocity distributions could be produced, allowing one to reach temperatures in the nanokelvin (nK) range.

**References**