Testing the impact of taxation on capacity choice: A ‘putty clay’ approach

Tarek M. Harchaoui, Pierre Lasserre

Abstract

We investigate the impact of taxation on a sample of Canadian copper mines based on a ‘putty clay’ model, i.e. focusing on plant creations and major capacity changes. Our theoretical analysis shows that the cost of capital differs in such an instance from its counterpart under a flexible neoclassical technology, particularly in the way the resource rent component enters the formula. Besides the influence of geological variables, our econometric results establish the effect of (net of federal, provincial, and mining taxes) expected prices on capacity choice. We complement these results by identifying a significant influence of tax characteristics, as distinct from the influence of gross prices per se.

Key words: Investment; Tax neutrality; Mining; Resource; Cost of capital

JEL classification: E22, H25, Q30

1. Introduction

This work studies the impact of taxation on the capacity choices of several Canadian non-ferrous mines. We assume that the technology is ‘putty clay’:

A good introduction to the analysis of taxation under various technology regimes is given by Bismar (1989).
ex ante, the firm has the flexibility to choose among many input combinations in order to maximize its objective function; ex post, once the optimal combination has been selected, the firm must use it, and the only flexibility that remains is the option either to operate at full capacity or to shut down. We also assume that the output price is characterized by a geometric Brownian motion.

This is indeed a very simple setup; but there is strong evidence in its favour and, considering the lack of reliable empirical results in this area, it is surprising that, to our knowledge, it has not been used in any econometric work on mines. Indeed, despite some earlier attempts and a strong interest by academics and professionals, some of the hypotheses tests and estimated magnitudes presented in this paper have no counterpart in the literature on mining taxation. We believe that many industries have characteristics similar to the one studied here, so that our approach could be useful in other areas. Firms in such industries are capital-intensive. Investments are bulky and basically irreversible. Prices are subject to wide fluctuations and basically out of the firm's control. Taxation and other kinds of public interventions are important, especially to the initial project development decision. There is a lot of anecdotal evidence that Canadian non-ferrous mines correspond to this description.

Some harder evidence is also available. An abundant literature tests Hotelling's rule, and fails to corroborate it; constant production, as implied by the 'putty clay' hypothesis is often suggested as an explanation in that literature (for numerous references, see Lasserre, 1991). More directly, in earlier work on some of the firms studied here, Lasserre (1985) used a model which allowed two phases in the life of a mine: ex ante, before any major investment, and ex post, once the plant (mine and concentrator) exists. By comparing rates of substitution ex ante and ex post, he was able to test whether, ex post, flexibility was unchanged (the 'putty putty' hypothesis), reduced (the 'putty semi putty' hypothesis of Fuss, 1977), or suppressed (the 'putty clay' hypothesis). The last possibility was the only one not to be rejected. This does not mean that decisions are never revised in reality; it does imply, however, that, as a simplifying assumption, the 'putty clay' hypothesis performs better than competing assumptions involving ex post price sensitivity of major quantity decisions.

Our assumption of a Brownian motion for expected real output price is a minor but empirically significant theoretical improvement over what remains the most common working assumption – perfect certainty – in studying the impact of taxation (especially corporate income taxation) on

\[\text{Bodie and Rozansky (1980) found the standard deviation of annual changes in copper prices over the period 1950–1976 to be 47.2 percent.}\]

\[\text{For a theoretical treatment of extraction under Brownian prices, see Pindyck (1980).}\]
neoclassical firms. This comment applies to conventional firms (Boadway, 1980) as well as extractive firms (Gaudet and Lasserre, 1984; Slade, 1984). While richer processes might be more adequate in general for resource prices, our data do not allow us to reject the assumption that real copper prices follow a Brownian motion over the period of our study; we do, however, investigate the sensitivity of our results to that hypothesis.

Because mining taxation is so much about the taxation of capital, the literature on capital and investment, as well as its ramifications on taxation per se, is relevant to our study. All major approaches to investment theory have been extended to the extractive firms: in particular, the case of perfectly malleable capital; the 'putty clay' model; the cost-of-adjustment model (see, for example, Lasserre, 1985; Gaudet, 1983). As far as taxation is concerned the perfect-malleability model has been extended to extractive firms (Gaudet and Lasserre, 1984, 1986) and formula for the calculation of marginal effective tax rates or the cost of capital have been worked out (Boadway et al., 1987, 1989) for various tax systems. While these calculations clearly show that taxation creates important price distortions, the direct impact of taxes, as distinct from gross prices, on production decisions has not been ascertained statistically, whether at firm level, or at more aggregate levels. In fact, most students of mining taxation (see, for example, Foley and Clark, 1984 and Campbell and Wrean, 1985) used simulations, most of the time based on a 'putty clay' model along the line initiated by Bradley et al. (1981). More recently, some authors have investigated investment and the impact of taxation within models borrowed from the financial literature on option pricing, which they apply to irreversible investment (e.g. Mackie-Mason, 1990). They focus on the value of waiting for new information, which we leave aside in the present paper and, rather than attempting, as we do, to infer the influence of taxes and other economic signals on the real magnitudes of investment projects, they rely on simulations. However, because our work shares several basic technological, and stochastic, assumptions with this new line of research, it could provide a basis for its extension beyond the realm of simulation into that of statistical inference.

Consequently, the econometric work that we are presenting, although conventional in many respects – it provides estimates of output-price and factor-price elasticities as well as measures of the impact of various tax parameters – fills an old gap and, as it leaves out the timing of investment decisions, points toward new research needs. Our hypotheses tests concern

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4 To a large extent, this comment also applies to all industries: while tax-adjusted prices, in particular the user cost of capital, are routinely computed and often found to have a significant impact on real decisions, we do not know of any paper establishing that taxes are non-neutral in a statistically significant way.
the influence of taxation on real decisions by mines. The profession claims, and most serious researchers believe, that the impact is significant; however, to our knowledge, our results constitute the first statistical evidence to that effect in resource sectors, and, in any sector, the first evidence that tax parameters may have a significant impact, distinct from that of pre-tax prices.

The remainder of the paper is organized as follows. In Section 2, we present and solve the theoretical model; while the 'putty clay' assumption facilitates the analysis by implying a constant output over the life of the mine, the optimizing decision still retains some complexity due to the fact that capacity will also determine the operating life of the firm, given its mineral reserves. After introducing the basic features of the model, we provide a brief description of mining taxation in Canada and we describe the way it is introduced into the model. Then, we discuss some economic aspects, focusing on the notions of scarcity rent and user cost of capital. Finally, we present the version of Hotelling’s lemma which applies to the model, and establish some key properties of factor demands and output supply. Section 3 is devoted to the description of the econometric work. This includes a description of the data, a sub-section on specification and econometric considerations, and two result sub-sections. The conclusion emphasizes major results, their implications, and their limits, including the need for future work.

2. The model

2.1. Setup

Let \( s \) be the start-up date for some firm. Our model focuses on decisions that are conditional on \( s \) having been selected. At \( s \), the firm knows with certainty its mineral reserves \( R(s) \) as well as the production function. The ore is transformed into metal or concentrate, whose nominal price at date \( t \) is \( p(t) \). Taxes being defined with reference to nominal prices, it is preferable to carry out the analysis in nominal terms. The real component of \( p(t) \) is assumed to follow a Brownian geometric motion so that

\[
\frac{dp_t}{p_t} = \alpha_p \, dt + \sigma_p \, dz_p ,
\]

where \( i_t \) is the rate of inflation; and \( dz_p \) is the increment of a Wiener process; \( dz_p = \epsilon(t)(dt)^{1/2} \), where \( \epsilon(t) \) is a serially uncorrelated and normally distributed random variable. This implies that the current value of \( p \) is known with certainty whereas future values are random and log-normally distributed around a trend (inflation plus drift) with a variance which grows
with the distance into the future. We assume that parameters $\alpha_p$, which reflects the drift of $p(t)$ in real terms, and $\sigma_p$, are constant. Under these assumptions (see, for example, Hull, 1992), the expected value of $p(t)$, conditional on information available at $s$, is

$$E_s(p(t)) = p(s)E_s\{I_{s,t}\} e^{\alpha_p(t-s)},$$

where $I_{s,t} = \exp(\int_s^t \sigma d\tau)$ is the inflation factor between $s$ and $t$ (with $I_{s,s}I_{t,s} = 1$) and $E_s$ is the conditional expectation operator.

The firm faces two kinds of costs: variable costs $w(t)L(s)$ incurred during its operating life, and a sunk nominal cost $q(s)K(s)$ paid instantaneously at $s$ where $w(t)'$ is the transposed vector of nominal variable factor prices at $t$, $t \in [s, \infty)$, and $q(s)$ is the price of one unit of the composite capital asset. Since we assume that the mining firm operates under a 'putty clay' technology, it has the flexibility to choose factor ratios and levels ex ante while, ex post, it must operate with the factor combinations selected at $s$. The sole difference between capital $K$, and variable factors $L$, in that context, is the fact that capital is paid fully at $s$ while variable factors are hired at the going price $w(t)$ during the entire operating life, and it is assumed that

$$E_s(w(t)) = w(s)E_s\{I_{s,t}\} e^{\alpha_p(t-s)}.$$  

We assume that the productive capacity of capital is maintained throughout the operating life at a cost included in the definition of variable costs. This may be modelled as follows. Output is given by a conventional production function $f(K, L - M(K), s)$ where $K$ is capital, $L$ is the vector of variable factors affected to either production or the maintenance of $K$, and $M(K)$ represents factors diverted from production activities in order to maintain capital. The level of technology is indexed by the date $s$. Then we define the production function, net of maintenance efforts, as $F(K, L, s) = f(K, L - M(K), s)$. Capital maintenance does not preclude economic, and tax, depreciation; in fact, at the closing date, capital does not retain any residual value. This is the most realistic simplifying assumption, as mining capital is highly firm-specific and difficult to use elsewhere. In the spirit of the 'putty clay' hypothesis, we assume that technological change affects production possibilities ex ante but not ex post, so that the time argument in $F$ is $s$ rather than $t$. Then production is constant at $Q$ during the whole operating life and the terminal date is

$$T = \frac{R(s)}{F(K(s), L(s), s)} = \frac{R(s)}{Q(s)}.$$  

In fact a significant proportion of capital consists of shafts and galleries which become valueless when operations cease.
Given a start-up date $s$, the objective of the firm is to choose, once and for all, its capital and variable-input mix so as to maximize its expected net present value subject to (2)–(4). In the absence of taxation, this is

$$W = E_x \int_s^T I_{t,s} e^{-rt} \{ p(t) F(K(s), L(s), s) - w(t)' L(s) \} dt - q(s) K(s),$$

where $r$ is the real discount rate, assumed constant over the operating period of the mine. Underlying the above objective function is the assumption that, once the initial investment has been made, it is not in the interest of the firm to close down, temporarily or permanently, before the full exhaustion of its reserves at $T$. This will indeed occur endogenously if, as we assume, net cash flows at all relevant future dates are such that (i) variable costs are covered at all dates and (ii) the output price is not expected to grow at such a rate that it might be desirable for the mine to hold on to its mineral reserves for speculative reasons. Both assumptions seem to be verified in practice for the firms in our sample, at least if we accept that the rate of growth in expected price cannot be wildly different from that of the actual price.

2.2. The Canadian tax system

Since the empirical part of the paper deals with Canadian firms, it will be useful and easier to approach taxation with the Canadian case in mind. A description of Canadian resource taxation over the relevant period is given in Holland and Kemp (1978) and Boadway et al. (1989); these pieces may be supplemented by sources in our data appendix. As a federal country, Canada raises taxes at the federal, the provincial, and the local, levels. Local taxes are not very significant for mines; we abstract from them, and focus on three taxes: the federal corporate income tax $FT$, the provincial corporate income tax $PT$, and the mining tax $MT$, also raised by the provinces. The constitution allocates jurisdiction on matters such as foreign and interprovincial trade to the federal government, while provinces have jurisdiction over natural resources. As these resources are mostly traded away from their provinces of origin, their taxation is bound to involve both levels of government. An important issue is the treatment of each tax in defining the bases of the other taxes. In particular, not recognizing mining taxes in the definition of the federal corporate income tax base would amount to denying provinces their constitutional jurisdiction over natural resources; but allowing full deduction exposes the federal government to possibly an unacceptable erosion of its base.

These considerations may explain some specific characteristics of the Canadian taxation system, as it applies to mines, and some of its reforms.
over our sample period. In fact, an interesting aspect of our study is the fact that the tax situation differs notably between observations, both because of location and because of tax reforms. The treatment of mining taxes by federal and provincial governments, and the depletion allowance, are important aspects of the taxation regimes. The depletion allowance is a deduction, analogous to capital depreciation allowances, corresponding to the natural-resource capital (reserves) used up during the period. In principle, the mining tax should reflect the value of current reserve depletion. Indeed, it was deductible from the federal and provincial corporate income tax bases in the first years of our sample period. Later on, in an attempt to disconnect the corporate income tax base from the mining tax, legislators disallowed the deduction of the mining tax, replacing it with a more generous depletion allowance. Whether or not mining taxes are deductible, the depletion allowance attempts to reflect the value of the resource forgone in the production process; it is defined as a proportion of the difference between revenues and allowable costs. If these costs coincide with the costs allowed in the computation of the corporate income tax base, the effect of the depletion allowance is simply a reduction in the corporate income tax rate. Such is the case in the example of a 1960 Quebec mine presented below.

Another particular feature of Canadian mining taxation is the presence, over some periods, of a tax holiday. The tax holiday is a period (usually 3 years) over which the firm does not pay federal or provincial corporate income taxes, although it pays mining taxes. As Mintz (1990) emphasizes, it is very beneficial to the firm if capital depreciation deductions may be postponed until the holiday expires; otherwise profits are reduced during the holiday, making it much less attractive. In our empirical application we allow for these alternative possibilities and define the present values of depreciation allowances accordingly.

A series of major reforms took place between 1972 and 1976, abolishing the tax holiday, abolishing the deductibility of mining taxes, and modifying the depletion allowance. The treatment of exploration expenditures was also modified, but our study abstracts from this aspect. Boadway et al. (1989) found that, in a period corresponding to the last years in our sample, “on the whole, the corporate and mining tax regimes in Canada provide a substantial subsidy to investments in the mining industry”... “Marginal effective tax rates are highly negative for many of the major expenditures undertaken by these mining firms.” In fact, Boadway et al. (1987) computed 1985 marginal effective tax rates in Quebec at between −98 percent and 30 percent, depending on the asset class. Tax-induced distortions in the price of capital relative to the price of output are shown in Fig. 1 below, for the mines in our empirical study, at the dates where major capacity investments were made. Values of one would characterize a neutral system. Clearly, the
tax system became much more favourable to capital as a result of the 1972–1976 reforms (right axis).

2.3. The model with taxes

We assume that firms expect the tax regime in place at $s$ to prevail during their whole operating life. Consider, as an example, a firm created in Quebec in 1960 (we will use the convention that 1960 is zero in that case). It benefits from a tax holiday of $T_1 (=3)$ years over which it pays the mining tax only, which is linear in profits:

$$MT = u_M(pQ - w'L - DC_M - DV_M), \text{ for } 0 \leq t \leq T_1,$$

where $u_M$ is the mining tax rate; $DC_M$ is the depreciation allowance corresponding to equipment, machinery, and structures; and $DV_M$ is the depreciation allowance corresponding to development expenditures. For subsequent years, the firm pays federal and provincial corporate income taxes, as well as the mining tax, for a yearly total of
\[ FT + PT + MT = u_F(pQ - w'L - DC_F - DV_F - DP_F - MT) + u_P(pQ - w'L - DC_P - DV_F - DP_P - MT) + u_M(pQ - w'L - DC_M - DV_M), \quad \text{for } t > T1, \quad (7) \]

where \( u_j \) is the tax rate under tax \( j = F \) (for federal), \( P \) (for provincial), \( M \) (for mineral), applied to the appropriate tax base. Each tax base is defined as revenues minus variable costs, minus allowable deductions. In general, these deductions fall under four major categories: depreciation allowances for equipment, machinery, and structures \( DC_j \); depreciation allowances related to development expenditures \( DV_j \); depletion allowances \( DP_j \); and deductible taxes. Of course, some of these deductions or allowances may not exist under one tax regime or the other. Thus, in the current example, \( MT \) (called mineral tax at the time) is deductible from both the federal and the provincial corporate income taxes and does not involve any depletion allowance.

Because they were defined as shares of the tax base, the depletion allowances of both the provincial and the federal systems amounted in 1960 to reducing the relevant tax rate \( u_j \) by a coefficient of \( (1 - a_j) \), where \( a_j \) is the deduction rate applying to the depletion allowance \( j \). Expressing \( DP_j \) accordingly in (7), and substituting for \( MT \), we can write the expected net present after-tax value of the Quebec 1960 mine, the sum of expected net revenues over the first \( T1 \) years (tax holiday) during which the mine pays only mineral taxes, and expected net revenues over the remaining period, when the firm pays all three taxes, as

\[
W = E_0 \left\{ \int_0^{T1} \left[ I_{t,0} e^{-r t} [p(t)Q(0) - w'(t)L(0)](1 - u_M) \right. \\
+ u_M(DC_M + DV_M)) \, dt \right\},
+ E_0 \left\{ \int_{T1}^{T} \left[ I_{t,0} e^{-r(t-T1)} \left[ (1 - u_F)[p(t)Q(0) - w'(t)L(0)] \\
+ u_F(1 - a_F)(DC_F + CV_F), \\
+ u_P(1 - a_P)(DC_P + DV_P) + u_M(1 - u_F(1 - a_F) \\
- u_P(1 - a_P)(DC_M + CV_M)) \right) \, dt \right\}, \\
- q(0)K(0), \quad (8) \]

where

\[ u_1 = u_F(1 - a_F)(1 - u_M) + u_P(1 - a_P)(1 - u_M) + u_M. \]
Depreciation allowances are measured in the standard way. As far as capital allowances are concerned, in tax regime \( j \), for each dollar of the corresponding asset base, the tax base is reduced by the allowed depreciation rate \( \theta_j^i \) on assets of type \( i, i = E \) (for material-equipment), \( M \) (for machinery), and \( S \) (for structures). The asset base will differ according to the type of depreciation: with geometric depreciation, it is the residual value at date \( t \), \( \Phi_j^i(t) \); with linear depreciation, the asset base is the acquisition value \( C_j^i \). Since our data do not include a breakdown of asset types, we call \( C_j^A \) and \( \Phi_j^A(t) \) the asset values corresponding to the total of \( E \), \( M \), and \( S \) (\( A \) for aggregate), and we use an aggregate depreciation rate

\[
\theta_j^A = \sum_i \theta_j^i \gamma_i^i,
\]

where \( \gamma_i^i \) is the share of asset type \( i \) in capital at the industry level. Capital depreciation allowances corresponding to the composite asset \( A \) under the mineral tax (they are of the linear type) are then \( DC_j^M = \theta_j^A C_j^A \) for as many years as depreciation takes place. As far as depreciation allowances for development expenditures are concerned, the same analysis applies, except that there is no need to aggregate over asset types since the law provides for one asset base only. For mineral tax purposes the development allowance is geometric, so that, using the same notation as above, it amounts to \( DV_j^M(t) = \theta_j^D \Phi_j^D(t) \) in all periods of the asset’s tax life. Similarly, for all capital or development allowances in (8),

\[
either DC_j = \theta_j^A C_j^A, \quad \text{or} \quad DC_j(t) = \theta_j^A \Phi_j^A(t),
\]

\[
and
either DV_j = \theta_j^D C_j^D, \quad \text{or} \quad DV_j(t) = \theta_j^D \Phi_j^D(t).
\]

Thus, the unique initial capital expenditure \( q(0)K(0) \) incurred by the firm at start-up time implies an array of future tax deductions. We do not investigate possible distortive effects of the tax system with respect to the mix of capital expenditures. Note that, at acquisition date, which we identify with start-up, \( C_j^A = \Phi_j^A(0) \) and \( C_j^D = \Phi_j^D(0) \) for all \( j \), so that total initial investment is

\[
q(0)K(0) = C_j^A + C_j^D = \Phi_j^A(0) + \Phi_j^D(0).
\]

We have now evaluated all components in (8). The standard approach to evaluating effective after-tax prices or effective tax rates, as summarized in Boadway (1980), then involves integrating the expected value function, which requires computing expected present values for capital and development allowances. In order to implement this approach here, we treat any geometric capital depreciation as if it continued forever as described, and we
treat any linear depreciation as if it was completed at $T$. This means that the
undeprcciated component at $T$ is either negligible or can be transferred to
other operations. Both possibilities are, in fact, a good approximation of
reality. It follows that the present value of depreciation allowances is
independent of $T$ and of any residual capital remaining at $T$. A further
complication has to do with the treatment of the allowances during the tax
holiday. In some tax systems, firms must claim them during the holiday, thus
diminishing their profits and reducing the attractiveness of the holiday; in
other instances (capital and development allowances for federal and provin-
cial purposes in our 1960 Quebec example), they may start being claimed
after the holiday has expired. In order to be able to make the appropriate
distinctions, for tax regime $j$ and asset $i$, let

$$
x_j^i = \int_0^{T_1} \theta^i_1 I_{r,0} e^{-rt} \, dt, \quad \bar{x}_j^i = \int_{T_1}^{1/\theta^i_j} \theta^j_1 I_{r,0} e^{-rt} \, dt,
$$

$$
\bar{x}_j^i = \int_{T_1}^{1+(1/\theta^j_1)} \theta^j_1 I_{r,0} e^{-rt} \, dt, \quad (11)
$$

be present values of straight line depreciation allowances associated with an
initial capital or development expenditure of one dollar. The first expression
gives the discounted sum of the deductions over the first $T_1$ years, when
depreciation starts being claimed during the tax holiday; the second
expression gives the value of the continuation after $T_1$ and until exhaustion
of the base, at $1/\theta^j_1$; the last expression covers cases where depreciation
allowance claims start after the tax holiday has ended. Similarly, for
geometric depreciation (omitting subscripts and superscripts)

$$
z = \int_0^{T_1} I_{r,0} e^{-(r+\theta)\theta t} \, dt, \quad \bar{z} = \int_{T_1}^{\infty} I_{r,0} e^{-(r+\theta)\theta t} \, dt,
$$

$$
\bar{z} = \int_{T_1}^{\infty} I_{r,0} e^{-rt} e^{-\theta(t-T_1)} \theta \, dt. \quad (12)
$$

Let us now substitute (9) and (10) into (8), distribute the expected value
operator under the integral terms, and divide the resulting expression into a
sum of integrals. The terms involving expected prices may then be inte-
grated using (2) and (3), while (11) and (12) may be used to eliminate
integrals\(^6\) in \(\theta_j^i\). Omitting time indices since all variables are evaluated at \(s = 0\)

\[
W = (1 - u_M) \left[ \left( 1 - \frac{e^{-\delta T_1}}{\delta} \right) \frac{pQ}{\rho} - \left( 1 - \frac{e^{-\rho T_1}}{\rho} \right) w'L \right] + u_M(\bar{z}_M C_M^A + \bar{z}_M D_M^D) \\
+ (1 - u_1) \left[ \left( \frac{e^{-\delta T_1} - e^{-\delta T}}{\delta} \right) \frac{pQ}{\rho} - \left( \frac{e^{-\rho T_1} - e^{-\rho T}}{\rho} \right) w'L \right] \\
+ u_f(1-a_f)(\bar{z}_f^A \Phi_f^A + \bar{z}_f^D \Phi_f^D) + u_p(1-a_p)(\bar{z}_p^A C_p^A + \bar{z}_p^D \Phi_p^D) \\
+ u_M[1 - u_f(1-a_f) - u_p(1-a_p)](\bar{z}_M^A C_M^A + \bar{z}_M^D \Phi_M^D) - qK,
\]

where \(\delta = r - \alpha_p\) and \(\rho = r - \alpha_p\). Remembering that, at start-up time, \(C_j^i = \Phi_j^i\), we can write \(C_j^i = \Phi_j^i = \gamma^i qK\), where \(\gamma^i\) is the proportion of asset \(i\) in initial capital expenditures. The capital allocation rule is obtained by setting the derivative of (13) with respect to \(K\) equal to zero:

\[
\left\{ \left[ (1 - e^{-\delta T_1}) A_1 + (e^{-\delta T_1} - e^{-\delta T}) A_2 \right] - \delta T A_2 \left( e^{-\delta T} - e^{-\rho T} \frac{w'L}{pQ} \right) \right\} pF_K = q\delta A_0,
\]

where \(F_K\) is the partial derivative of \(F\) with respect to \(K\),

\[
A_0 = 1 - u_M(\gamma^A \bar{z}_M^A + \gamma^D \bar{z}_M^D) - u_f(1-a_f)(\gamma^A \bar{z}_f^A + \gamma^D \bar{z}_f^D) \\
- u_p(1-a_p)(\gamma^A \bar{z}_p^A + \gamma^D \bar{z}_p^D) \\
- u_M[1 - u_f(1-a_f) - u_p(1-a_p)](\gamma^A \bar{z}_M^A + \gamma^D \bar{z}_M^D),
\]

\[
A_1 = 1 - u_M,
\]

and

\[
A_2 = 1 - u_1.
\]

Each observation of a mine at its start-up date is described in a similar way as the 1960 Quebec mine just characterized. The formulas change depending on the tax regimes and so do parameter values, but the approach remains unchanged.

\(^6\) Our assumption on depreciation horizons makes this elimination possible as it allows the identification of terms whose upper bound is infinite in (12), with the corresponding terms in \(W\), whose upper bound is \(T\).
2.4. Resource rent and capital allocation

Let us bring up the simple economics which underlie expression (14). The most important term is $A_0$, which is a complicated combination of two types of tax parameters: tax rates $u_j$ and pseudo tax rates $a_j$ on one hand, and parameters defining various deductions from taxable income (the $z$'s and $x$'s) on the other hand. In the absence of taxation, or if the combined tax regimes are neutral in the sense that they leave the cost of capital unchanged, then $A_0 = 1$. Otherwise, $A_0$ is normally smaller than one, perhaps even negative. A $A_0$ is small if two conditions are met: the deductions for capital depreciation and development expenditures are high enough (high $z$'s and $x$'s) to reduce the tax bases substantially; the tax rates are high enough to make such reductions in the bases financially interesting. Indeed (15) illustrates the fact that higher tax rates distort factor allocation in favour of capital if depreciation and similar deductions are important. However, the favourable impact on $K$ which may result from a low cost of capital on the right-hand side of (14), may be mitigated by the impact of taxation on the left-hand side. On that side, which measures the after-tax value of marginal product, we identify three tax parameters or groups of parameters, none of which includes deductions for capital expenditures or assimilated provisions: $A_1$, $A_2$, and $T1$. The expression $(1 - A_1)$ is the effective tax rate rate when there is a tax holiday (whose duration is then $T1$); $(1 - A_2)$ is the effective tax rate after the expiration of the tax holiday, and whenever there is no such provision. A value of one for either $A_1$ or $A_2$ means that the firm keeps one dollar of after-tax income for each dollar of pre-tax income; lower values indicate that the tax bites into income. To the extent that $A_1$ is more favourable than $A_2$, long tax holidays are desirable to the firm. Another remarkable feature, true of all tax regimes in our sample, is that they do not affect the price of variable factors relative to the output price.

How does the investment decision rule under the ‘putty clay’ model compare with its counterpart with malleable capital? The comparison will be easier if we abstract from taxation issues for a while, by setting $A_0 = A_1 = A_2 = 1$. Under a flexible neoclassical technology the capital allocation rule is (Gaudet and Lasserre, 1986)

\[
(p_t - \lambda)F_K = q_t(r_t + \theta) - \frac{dq_t}{dt}.
\]  

7 As mentioned earlier, although $a_j$ actually characterizes a deduction, the depletion allowance, its effect is simply to modify the tax rate.

8 A negative value of $A_0$ implies that the after-tax cost to the firm of the marginal unit of capital is negative, a situation which is not considered unusual in extractive sectors.
This is a version of the well-known rule which leads to choosing capital in such a way that the value of its marginal product (the left-hand side) be equal to its rental rate (the right-hand side). Note that the unit value of the marginal product, \( p_t - \lambda_t \), includes a term, the scarcity rent \( \lambda_t \), which is specific to the extractive firm and reduces net output price by the opportunity cost of the resource. The corresponding capital allocation rule under a 'putty clay' technology, obtained by setting \( A_0 = A_1 = A_2 = 1 \) in (14), is

\[
\left\{ p_s (1 - e^{-\delta_s T_s}) - \delta_s T_s \left[ p_s e^{-\delta_s T_s} - \frac{w'_s L_s}{Q_s} e^{-\rho T_s} \right] \right\} F_K = q_s \delta_s .
\] (19)

The first obvious implication of the 'putty clay' hypothesis is that (19) does not apply at all dates but only at start-up time. As in the malleable-capital case the left-hand side can be recognized to give the marginal product value, while the right-hand side gives the rental rate of an asset whose price does not change. The capital gain component, present on the right-hand side of (18), is absent from (19), as equipment is acquired and financed once and for all at start-up time. On the other hand, the future evolution of output prices does not matter when capital is perfectly malleable so that, in (18), the opportunity cost of funds need not be corrected for future capital gains or losses on the product side; this is not so with irreversible capital, where the real discount rate \( r \) is replaced with the adjusted discount rate \( \delta \) for that reason. Also, the depreciation term on the right-hand side of (18) is not present on the right-hand side of (19). This is only a difference in notation, due to our assumption that capital is maintained at a cost during operating life, with the corresponding expenditures being included in the definition of \( F \).

Focusing now on the left-hand side of (19), the coefficient of \( F_K \), which expresses, in rental units, the average value of the marginal product of capital, is also quite different. It is divided into two components. The first one gives the gross value of \( F_K \); it would be \( p_s \) if the extraction period was infinite; the fact that \( p_s \) is multiplied by \( (1 - e^{-\delta_s T_s}) \) reduces its value to reflect the finiteness of the extraction period. The second component corresponds to the scarcity rent which must be subtracted from the gross value. Its interpretation is as follows: for each marginal increase in \( K \), production will be higher by \( F_s \) over \( T_s \) periods; thus \( T_s F_s \) measures by how much reserves will be reduced at \( T \) because of the marginal increase in \( K \) at \( s \); the corresponding loss in income will not occur until \( T_s \), when the mine runs out of reserves; its expected value per unit will then be \( E_s (p_{T_s} - E_u (w'_{T_s} L_s / Q_s)) \), or \( \delta_s \left[ p_s \exp(-\delta_s T_s) - (w'_s L_s / Q_s) \exp(-\rho T_s) \right] \) in discounted flow units. An interesting difference between both (18) and (19), and their counterparts applying to conventional firms, is that a rise in the discount rate has an ambiguous effect on \( K \) (Lasserre, 1985), explained by the fact
that the effect of such a rise is not only to increase the cost of capital, but also to reduce the resource rent.

2.5. Properties of the theoretical model

The optimized value function corresponding to (13), evaluated at \( s \), is

\[
W^*(p, w, q, \delta, \rho, T1, R, A_0, A_1, A_2, s) = \left[ \frac{(1 - e^{-\delta T1})}{\delta} pQ^* - \frac{(1 - e^{-\rho T1})}{\rho} w'L^* \right] A_1 \\
+ \left[ \frac{(e^{-\delta T1} - e^{-\delta T})}{\delta} pQ^* - \frac{(e^{\rho T1} - e^{-\rho T})}{\rho} w'L^* \right] A_2 - qK^*A_0,
\]

(20)

where \( Q^*, L^* \), and \( K^* \) are functions of \( p, w, q, \delta, \rho, T1, R, A_0, A_1, A_2, \) and \( s \) that maximize (13), and \( T^* = Q^*/R \) is assumed at least equal to \( T1 \). Since \( W^* \) is an intertemporal profit function, applications of the envelope theorem give expressions analogous to Hotelling's lemma; in particular:

\[
\frac{\partial W^*}{\partial \rho} = \left[ \frac{(1 - e^{-\delta T1})}{\delta} A_1 + \frac{(e^{-\delta T1} - e^{-\delta T})}{\delta} A_2 \right] Q^* = \Sigma^* ,
\]

(21)

\[
\frac{\partial W^*}{\partial w} = \left[ \frac{(1 - e^{-\rho T1})}{\rho} A_1 + \frac{(e^{\rho T1} - e^{-\rho T})}{\rho} A_2 \right] L^* = -I^* ,
\]

(22)

\[
\frac{\partial W^*}{\partial q} = -K^*A_0 ,
\]

(23)

The model has the following properties, proven in Appendix A.

**Properties**

(i) \( W^* \) is convex in \( p, w, q, A_0, A_1, A_2 \).

(ii) \( W^* \) is homogeneous of degree one in \( p, w, q \), and homogeneous of degree one in \( A_0, A_1, A_2 \).

(iii) \( \Sigma^* \) is homogeneous of degree one in \( p, w, q, A_0, A_2 \); \( Q^* \) is homogeneous of degree zero in \( p, w, q \), and in \( A_0, A_1, A_2 \).

(iv) \( \partial Q^*/\partial \rho \geq 0; \partial Q^*/\partial A_1 \geq 0; \partial Q^*/\partial A_2 \geq 0 \) if \( T^* > T' \) (\( T^* < T' \)), where \( T' > T1 \) is defined in Appendix A; \( \partial Q^*/\partial T1 \geq 0 \) if \( A_1 = A_2 \); \( \partial Q^*/\partial q \) may be positive or negative (but \( \partial K^*/\partial q \leq 0); \partial Q^*/\partial w, \partial Q^*/\partial \delta, \partial Q^*/\partial \rho \) may be positive or negative; \( \partial Q^*/\partial R \geq 0 \) if \( K^* \) does not increase as \( Q^* \) increases.

The above properties are not exhaustive; they have been highlighted with two purposes in mind. First, a comparison with analogous, one-period
models, with or without taxes; second the empirical application. From both points of view the homogeneity properties are unusual, although less strikingly so in the case of $Q^*$ than in the case of $W^*$ and $\Sigma^*$. While $Q^*$ is the object of our empirical application, mines are so capital-intensive that one should expect the partial derivatives of $Q^*$ to have the same signs as the partial derivatives of $K^*$; this is why, in case of ambiguity of the effect of one variable on $Q^*$, we also checked whether its effect on $K^*$ was also ambiguous. We now turn to the empirical application.

3. Econometric model and empirical results

3.1. Data and alternative assumptions on price expectations and the discount rate

Although (21)–(23) could be used as a basis for an econometric model, our data set (we do not have reliable data on variable factors or capital equipment) allows us to use only capacity supply equation (21). Thus we want to estimate a single equation model giving $Q^*$ as a function of $p, w, q, \delta, \rho, T1, R, A_0, A_1, A_2,$ and $s$ that satisfies the relevant elements of the properties above. Inspecting the first-order conditions for $K$ and $L$ that underlie capacity choice, one verifies that there are natural combinations under which the explanatory variables may be introduced. Consider (14): the right-hand side $q \delta A_0$ is one such combination, whose economic interpretation is the after-tax cost of capital. While $q$ and $A_0$ enter the model only as part of $q \delta A_0$, $\delta$ may affect $W^*$ by another channel, as it enters discount coefficients. Consequently, it is natural to select both $WK = q \delta A_0$, and $\delta$ as explanatory variables. Similarly, one notes that $A_1, A_2,$ and $p$ appear either as $P1 = pA_1$, or as $P2 = pA2$. These are again natural choices as explanatory variables representing after-tax output prices, respectively during, and after, the tax holiday. The system of first-order conditions for $L$, although not as compelling as (14), suggests that variable-factor prices should preferably appear as $wAi/p, i = 1, 2$. However, distinguishing between after-tax, variable-factor prices during, and after, the tax holiday would take six variables instead of three, as opposed to two variables instead of one in the case of $p$; also, the distinction would imply that, in total, four variables be multiplied by $A1$, and four by $A2$, increasing risks of collinearity. Thus, we use pre-tax, variable-factor prices: $w/p$ is decomposed into $WL, WE,$ and $\ldots$
WM, corresponding respectively to the prices (divided by $\rho$) of labour, energy, and materials. The other determinants of $Q^*$, namely $\rho, R, T1$ and $s$, are introduced independently.

Since our sample size does not allow us to use a second degree flexible form, we use linear forms without cross effects. This does not preclude monotonic transformations of the variables, in particular logarithmic or semi-logarithmic forms. Model selection is then based on diagnostic tests, econometric results, and economic criteria. The fact that our specifications do not allow cross effects has one important consequence which can be dealt with by a slight alteration of the data set. When there is a tax holiday ($T1 > 0$), the relevant after-tax price is $P1$ over the first $T1$ years; hence the presence of both $P1$ and $P2$ in the specification; however, when $T1 = 0$, $P1$ is irrelevant, although well defined. We handle this contradiction by setting $P1 = 0$ whenever $T1 = 0$. As a result, $T1$ and $P1$ become somewhat collinear and $T1$ may be redundant as $P1$ now indicates both the presence, and the attractiveness, of the tax holiday.

The data set is made of individual observations on Canadian copper mines. An observation is the occurrence of a major capacity investment, which we define as either the creation of a new operation, or any capacity increase exceeding 20 percent of existing capacity. The data set is neither a time-series, nor a cross-section, nor a panel: there may be several observations in a given year; there may be years without observations; but date and location are important characteristics of any observation. The sample period covers the period 1960–1980. Fourteen different firms, some of which were observed more than once, make up a total of 23 observations, out of which 20 are located in British Columbia, 2 in Ontario, and 1 in Quebec. Note also that some of the variables, prices typically, are common to all firms and vary only according to the date, while others such as mineral reserves are mine-specific. Although output prices are common at any given date, after-tax prices are mine-specific because of the tax parameters entering their definition. Most of the firm-specific data are taken from various editions of the Canadian Mines Handbook; common data are taken from various aggregate sources (See Appendix B for details). General tax information comes from Holland and Kemp (1978); the effect of inflation is ignored in the computation of tax parameters. Specific details are given in Appendix B.

Two of the variables, $\delta = r - \alpha_r$, and $\rho = r - \alpha_w$ are not directly available. $\alpha_r$ is the real component of the drift in the random walk process (1). Before

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10 The data set is available on request.

measuring it, we want to test the random walk hypothesis. Expressing (1) in real Canadian dollars, Ito-integrating between \( t \) and \( t - 1 \), and then taking the logarithm of the result gives,

\[
\ln P_t = \alpha_0 + \ln P_{t-1} + \omega_t ,
\]

where \( P_t \) is the real price. If (1) is true, \( \omega_t \) is independently and identically distributed with variance \( \sigma^2 \), and \( \alpha_0 = \alpha_p - (1/2)\sigma^2 \). We test the random walk hypothesis by testing \( \alpha_1 = 1 \) in the augmented model below (Perron, 1992)

\[
\ln P_t = \alpha_0 + \alpha_1 \ln P_{t-1} + \alpha_2 t + \omega_t .
\]

The model is estimated for each of the 14 capacity creation dates, from the time-series of copper up to \( s \) (starting in 1940). Whether we use Dickey–Fuller’s (1981) unit root test, or a \( t \)-test of the \( \alpha_1 = 1 \) hypothesis, we find that the random walk hypothesis cannot be rejected. Thus firms are likely to form their price expectations on that basis, although alternative possibilities may be considered (see below).

There are several ways to evaluate \( \alpha_p \) and \( \delta \). One possibility is to use the estimates of \( \alpha_0 \) and \( \sigma_p \) from (constrained) model (25) to evaluate \( \alpha_p = \alpha_0 + (1/2)\sigma^2 \). Although not significantly different from 0, the resulting estimate of \( \alpha_p \) is in the order of 20 percent, much higher than the average yearly growth rate of less than 2 percent experienced by \( P \) over sample period. Thus, we choose the more widely used historical rate approach (e.g. Malliaris and Malliaris, 1992): for each \( s \), we evaluate \( \alpha_p \) as the average rate of growth over the period 1940–\( s \) (thus, it depends on \( s \)), and then we compute the adjusted real rate of return \( \delta \) under the random walk hypothesis as \( r - \alpha_p \).

The assumption of constant real price expectations, \( \alpha_p = 0 \), is also compatible with the results from estimating (25) (\( \alpha_0 \) and \( \alpha_2 \) not significantly different from zero). We generate the corresponding data by setting \( \delta = r \). Another alternative assumption, also compatible with the above statistical results is the constant adjusted discount rate assumption. Under that assumption, \( \delta \) is a constant and the corresponding data set can be generated by redefining \( W_k \) as \( qA_0 \) instead of \( \delta qA_0 \). Yet another alternative hypothesis is the adaptive price expectation hypothesis, whereby, beyond \( s \),

\[\text{See Kamien and Schwartz (1991, p. 266).}\]

\[\text{According to Perron (1992), Monte Carlo experiments indicate that the } t \text{-test is more reliable. Out of 14 regressions, the estimated value of } \alpha_1 \text{ varies between 0.64 and 0.72, with } t \text{-statistics for tests of } \alpha_1 \neq 1 \text{ ranging between 1.28 and 1.34. The estimates for } \alpha_0 \text{ are never significantly different from 0, while } \alpha_2 \text{ is never significantly different from 0 at the 5 percent level, but is positive at the 10 percent level of significance in 7 instances. It is interesting to note that, when prices are expressed in US dollars, rather than Canadian dollars, the random walk hypothesis is rejected.}\]
expected prices are assumed to move at some constant rate from a base level that depends on past prices up to \( s \). We postulate such a base level to be generated by an Almon process, which we arbitrarily define over a six-year period. The corresponding data set is obtained by setting \( p_s \), for each observation, as its estimated value at \( s \) from the Almon price model estimated over the period 1940–\( s \).14

With respect to \( p_i \), we consider three alternatives. The first alternative is to assume that firms expect variable-factor prices to follow the same trend as output prices over their operating life. This assumption can be justified by the fact that unions watch operating profits so that, over the long run, wages adjust to output prices. Under this assumption, \( p_i = \delta_i \), so that \( p_i \) is removed from the equation. The second alternative, our preferred option, is to assume that \( p_i = r_i \), as if firms expected constant future real factor prices. Under this assumption, \( r_i \) is substituted for \( p_i \) in the equation. The third alternative is to assume that \( p_i \) is the same at all mine creation dates. Under this assumption, \( p_i \) equals \( p \), and cannot be disentangled from the coefficients of variable factor prices and the constant. While they imply the same equation, the first and the third alternatives are not observationally equivalent, as variable-factor prices are divided by different values of \( p \).

3.2. Specification, econometric considerations, and theoretical restrictions

We experimented with linear, logarithmic, and semi-logarithmic forms of the capacity demand equation. An advantage of the semi-logarithmic form over alternative forms, is the fact that the residuals from its OLS estimation pass the Bera and Jarque (1980) normality test. Another advantage is that homogeneity of degree zero in \( A_0, A_1 \), and \( A_2 \) is not rejected for that form, although homogeneity in \( p, w, q \) is rejected. Thus, our reported results are based on variants of the following semi-logarithmic equation

\[
\ln(Q_{i,s}) = \theta_0 + \theta_{p1} P1_{i,s} + \theta_{p2} P2_{i,s} + \theta_{wK} WK_{i,s} \\
+ \theta_{rK} R_{i,s} + \theta_{WM} WM_{i,s} + \theta_{W1} WL_{i,s} + \theta_{W2} WE_{i,s} \\
+ \theta_{\delta} \delta_i + \theta_{p} p_i + \theta_{T1} T1_{i,s} + \theta_{s} s + \mu_{i,s}, \tag{26}
\]

where, to briefly recap the notation introduced earlier, \( P1 \) and \( P2 \) represent

14 The estimated models are, over the 1940s period,

\[
p_i = \alpha + a_0 X_{i,s} + a_1 X_{i,s} + a_2 X_{i,s} + u_i, \quad X_{i,s} = \sum_{j=1}^{n} p_{i-j},
\]

where \( u_i \) is an i.i.d. error term. In a representative case, based on the period 1940–1970, the corresponding estimated coefficients of \( p_{i-j} \), obtained as \( \beta_j = \alpha_i + a_j + a_j^2 \) were: \( \beta_1 = 0.234 \) (\( t = 4.85 \)); \( \beta_2 = 0.356 \) (\( t = 5.30 \)); \( \beta_3 = 0.368 \) (\( t = 6.21 \)); \( \beta_4 = 0.267 \) (\( t = 8.15 \)); \( \beta_5 = 0.056 \) (\( t = 0.87 \)); \( \beta_6 = -0.267 \) (\( t = -1.70 \)). The \( R^2 \) was 0.75. Detailed results are available on request.
after-tax output prices, respectively during, and after, the tax holiday; \( WK \) is the after-tax price of capital; \( R \) is the quantity of mineral reserves, \( WM, WL, \) and \( WE \) represent (gross of tax) variable-factor prices, respectively for materials, labour, and energy, divided by the common (adjusted) rate of discount \( \rho \) applicable to variable expenditures; \( \rho (= r - \alpha_w) \) also appears as a separate variable, together with \( \delta (= r - \alpha_p) \), the adjusted rate of discount applicable to revenues; \( T1 \) is the duration of the tax holiday; \( s \) is the observation date; and \( \mu \) is an independently, normally, distributed error term. \( i \) and \( s \) subscripts respectively refer to location and date.

Each alternative output price, or factor price, expectation assumption implies estimating (26) or its variants with a different data set, as described above. Similarly, the homogeneity assumptions may be imposed, in the case of tax homogeneity, by dividing each tax variable by \( A \); and, in the case of price homogeneity, by dividing each price variable by \( WM \). Consequently, these alternative hypotheses do not constitute restrictions of a given model, but involve different models; thus we test them using the \( J \)-test of Davidson and MacKinnon (1981).\(^{15}\)

Since there are many idiosyncrasies not taken into account by our explanatory variables, a substantial portion of capacity might be left unexplained by the model. If so, one might suspect that the errors in the model are heteroscedastic, perhaps also spatially correlated. Consequently, our reported inference tests are based on White (1980)'s consistent variance–covariance matrix. Considering the structure of our data bank, the Durbin–Watson statistic, which is based on the assumption that the data are built according to a certain order such as time or contiguity, is meaningless. However, it is plausible to believe that there is some correlation between certain error terms. For example, there may be ‘good’ years, and ‘bad’ years, characterized by different error terms. We introduced a dummy taking the value one if there was more than one observation that year (a good year), and zero otherwise. Its coefficient was not significant. Error terms might also vary systematically with the date of the observation; the fact that \( s \) is an explanatory variable, and turns out not be significant, implies that this type of error term correlation is not likely to be a problem here. The type of exploitation, open-pit or underground, is known to imply substantial differences in techniques and organization. The dummy variable \( EXPL \) which takes the value one for open-pit mines, was significant in most of our estimations. We also tried an alternative dummy variable, \( PROV \), taking value one for mines in British Columbia (20 observations) and zero for firms in the East (3 observations); regional and geological characteristics

\(^{15}\)The exact procedure used to carry out \( J \)-tests is described in Subsection 3.4 below. Homogeneity may be imposed as a parameter restriction (tested with an \( F \)-statistic) in log–log forms, but not when a semi-logarithmic form is used.
being often the same as the ones that govern the choice between open-pit, and underground, mining, that variable appeared to play the same role as EXPL but had less explanatory power. The inclusion of either one of these two dummies also constitutes a way to control for any spatial correlation which might be present in the model (Case, 1991).

Finally, we suspected that collinearity problems between some tax variables might explain the lack of robustness of some parameter estimates. Although the problem was not obvious from the correlation table, it is evident from Fig. 1, which shows a dramatic drop in $A_0/A_2$ occurring simultaneously with the removal of the tax holiday. This observation is reinforced by explicit statements (Holland and Kemp, 1978) to the effect that the tax holiday was traded against more favourable deductions in the tax reform series of 1972–1976. We substituted a dummy variable, REF, for $T1$ and (or) $P1$ in some regressions, REF, normally equal to zero, takes up a value of one whenever the tax treatment is so favorable as to cause $WK$ to be negative; this happens in all years after 1974.

3.3. Results and hypotheses tests

The results of estimating several variants of (26), using the data sets corresponding to the alternative output price expectations and discount rate assumptions described above, are presented in Table 1. Table 1 corresponds to our preferred assumption on factor prices: $\rho_f = r_f$. The results corresponding to alternative assumptions – $\rho_f =$ constant and $\rho_f =$ $\delta_f$ – are presented in Table 3, available on request. Each column gives the results of estimating a particular equation. $r$ and $s$ are not in the list of explanatory variables because their coefficients were never significant. This does not mean that $r$ does not influence capacity decisions but that the major part of its influence occurs via variable-factor prices. Eqs. (27)–(30) correspond to constant expected real output prices (as described earlier, the associated data set is constructed with $\alpha_p = 0$); (27) and (28), which differ only in their constant terms, are based on the static expectation formation hypothesis (prices on which expectations are based are the observed copper prices at each investment date); (29) and (30) are identical in specification to (27) and (28), but based on the adaptive expectation hypothesis (prices on which expectations are estimated, at $s$, by an Almon lag model). Both pairs of equations yield very similar results, implying a low sensitivity to the way history affects the initial level of the expected price path. In fact, $J$-tests of the adaptive expectation formation hypothesis ((29) or (30)) against the static expectation formation hypothesis ((27) or (28) respectively) are inconclusive. This also holds true for models with non-constant price expectations, which make up the rest of Table 1 although, in their case, we
Table 1

Capacity choice: Alternative output-price expectations – $\rho = r^n$

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_p = 0$</th>
<th>$\alpha_p = 0$</th>
<th>$\delta = constant^b$</th>
<th>$\alpha_p =$ historical rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Static initial price</td>
<td>Adaptive initial price</td>
<td>Static initial price</td>
<td>Static initial price</td>
</tr>
<tr>
<td></td>
<td>(27)</td>
<td>(28)</td>
<td>(29)</td>
<td>(30)</td>
</tr>
<tr>
<td>CONS</td>
<td>-0.481</td>
<td>0.379</td>
<td>1.226</td>
<td>1.584</td>
</tr>
<tr>
<td></td>
<td>(-0.666)</td>
<td>(0.574)</td>
<td>(0.980)</td>
<td>(1.833)</td>
</tr>
<tr>
<td>P1</td>
<td>0.280</td>
<td>0.275</td>
<td>0.432</td>
<td>0.322</td>
</tr>
<tr>
<td></td>
<td>(1.079)</td>
<td>(1.049)</td>
<td>(1.100)</td>
<td>(0.880)</td>
</tr>
<tr>
<td>WK</td>
<td>-0.393</td>
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<td>0.704</td>
</tr>
<tr>
<td></td>
<td>(-1.078)</td>
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<td>(0.548)</td>
<td>(1.404)</td>
</tr>
<tr>
<td>WL</td>
<td>1.873</td>
<td>2.451</td>
<td>3.641</td>
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</tr>
<tr>
<td></td>
<td>(2.745)</td>
<td>(4.983)</td>
<td>(0.654)</td>
<td>(0.451)</td>
</tr>
<tr>
<td>WE</td>
<td>-0.082</td>
<td>-0.258</td>
<td>-0.659</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(-0.227)</td>
<td>(-1.595)</td>
<td>(-0.543)</td>
<td>(1.162)</td>
</tr>
<tr>
<td>WM</td>
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<td>-1.641</td>
<td>-2.892</td>
<td>-0.238</td>
</tr>
<tr>
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<td>(-5.531)</td>
<td>(-0.649)</td>
<td>(-0.260)</td>
</tr>
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<td>T1</td>
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<td>0.942</td>
<td>0.393</td>
</tr>
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<td></td>
<td>(6.684)</td>
<td>(6.626)</td>
<td>(0.946)</td>
<td>(0.991)</td>
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<td>REF</td>
<td>-0.967</td>
<td>4.720</td>
<td>-0.174</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(-1.336)</td>
<td>(0.620)</td>
<td>(0.620)</td>
<td>(0.620)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R$</td>
<td>$0.749 \times 10^{-8}$</td>
<td>$0.760 \times 10^{-8}$</td>
<td>$0.943 \times 10^{-8}$</td>
<td>$0.698 \times 10^{-8}$</td>
</tr>
<tr>
<td>EXPL</td>
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<td>2.190</td>
<td>1.966</td>
<td>2.022</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.93</td>
<td>0.92</td>
<td>0.92</td>
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<tr>
<td>$R^2$ adj.</td>
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<td>0.88</td>
<td>0.85</td>
<td>0.86</td>
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<tr>
<td>$F^c$</td>
<td>16.5</td>
<td>18.6</td>
<td>13.3</td>
<td>15.7</td>
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<tr>
<td>$B-J^d$</td>
<td>0.27</td>
<td>0.49</td>
<td>0.47</td>
<td>0.39</td>
</tr>
</tbody>
</table>

*a*-statistics between parentheses.

*b* When $\delta = constant$, WK is defined as $qA_0$ rather than $\delta A_0 q$.

$c$-F-statistics defined from mean.

$d$-Bera-Jarque asymptotic Lagrange multiplier normality test.
do not report the results of estimating the adaptive expectation formation versions of the models.

Eqs. (31) and (32) correspond to the constant adjusted discount rate hypothesis: since $\delta$ is constant, it does not appear as an individual variable, and $WK$ is redefined as $qA_0$ instead of $\delta qA_0$. For these two equations, the value of $\delta$ is buried into the coefficient of $WK$ and into the constant so that it cannot be recovered; this does not matter since we are interested in result sensitivity and model comparisons rather than parameter estimates at this stage. Finally, (33) and (34) correspond to the model described in the theoretical section of the paper, based on the random walk hypothesis.

All reported models pass the Bera-Jarque normality test, giving us some confidence in the reliability of reported $t$-tests, despite the low number of observations. Considering that they are cross-section-like models, the explanatory power is high, as indicated by the $R^2$ and $F$ statistics. $J$-tests (unreported) allow us to reject (27) in favour of (33) and (34), and (31) in favour of (33) and (34), confirming the initial impression from basic statistics in favour of the random walk hypothesis. Besides the Bera–Jarque test passed at a better (lower) than 80 percent significance level, (34) was submitted to Ramsey's RESET test which rejected the presence of any statistically significant correlation between the residuals and the calculated dependent variable. From an economic point of view, the three groups of models give qualitatively similar results (signs are identical when parameter estimates are significant) but (34) has more significant individual coefficients. Price homogeneity is rejected for all reported models but tax homogeneity cannot be rejected. However, the Bera–Jarque normality test reaches the 90 percent significance level in the (unreported) tax homogeneous version of (34). Besides the statistical and economic superiority of (34), we prefer the random walk hypothesis because, while many other expectation formation assumptions might approximate reality closer, few, if any, lend themselves to the simple and coherent theoretical analysis presented in the first part of the paper. To recap briefly, (34) is based on the assumptions that $p_r$ follows a random walk in real terms, and that $p_r$ may be identified with $r_r$. Before proceeding to further analysis, it should also be noted that comparisons of estimate magnitudes across columns are difficult because, as described above, the data sets associated with each group of models may differ in output price, factor prices and adjusted discount rate.

The main focus of the foregoing empirical work is the influence of taxation on capacity choice. Further statistical tests will be useful in order to complete this aspect of the analysis. Before presenting them, let us discuss other economic and technological implications of our results. We give the results in elasticity form in Table 2, for (34). These elasticities are computed at mean sample values, and also at the sample maximum and minimum
Table 2
Elasticity estimates: Model (34)

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon_{Q,P}$</th>
<th>$\epsilon_{Q,AT}$</th>
<th>$\epsilon_{Q,AT}$</th>
<th>$\epsilon_{Q,W}$</th>
<th>$\epsilon_{Q,W}$</th>
<th>$\epsilon_{Q,WE}$</th>
<th>$\epsilon_{W,WE}$</th>
<th>$\epsilon_{Q,\beta}$</th>
<th>$\epsilon_{Q,\gamma}$</th>
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</thead>
<tbody>
<tr>
<td>With</td>
<td>Without</td>
<td>With</td>
<td>Without</td>
<td>With</td>
<td>Without</td>
<td>With</td>
<td>Without</td>
<td>With</td>
<td>Without</td>
</tr>
<tr>
<td>tax</td>
<td>tax</td>
<td>holiday</td>
<td>tax</td>
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<td>tax</td>
<td>holiday</td>
<td>tax</td>
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<tr>
<td>min</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>1.904</td>
<td>1.766</td>
<td>0.417</td>
<td>0</td>
<td>1.990</td>
<td>-1.234</td>
<td>0.0026</td>
<td>2.377</td>
<td>-0.371</td>
</tr>
<tr>
<td>mean</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.093</td>
<td>2.870</td>
<td>0.468</td>
<td>0</td>
<td>2.870</td>
<td>-0.352</td>
<td>0.952</td>
<td>4.147</td>
<td>-0.958</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.512</td>
<td>6.968</td>
<td>0.491</td>
<td>0</td>
<td>3.138</td>
<td>-1.460</td>
<td>6.907</td>
<td>11.689</td>
<td>-5.177</td>
</tr>
</tbody>
</table>

- p-values between parentheses.
- When a variable is negative at its sample minimum the elasticity is inverted, so that its sign reflects the direction of the relationship between capacity and the variable.
values of the variable under study. There is an important conceptual difference between Table 1 and Table 2. Some variables affect capacity both directly, and via their impact on other variables. Thus has an impact of its own, and also enters the formula giving WK. Table 1 gives the coefficient associated with the partial, direct, effect, while Table 2 gives the total elasticity. Thus, for example, the formula for the elasticity of Q with respect to is not simply but

\[ e_{Q,\delta} = (\theta_\delta + \theta_{WK} q A_0) \delta. \] (35)

For such variables, the t-statistics differ between Table 1 and Table 2, while they are identical in the case of variables whose sole effect on Q is direct. In the case of , although does not enter (34) directly, the elasticity is reported because enters the definition of all three variable factor prices; furthermore, while the coefficients associated with each variable factor price are highly significant, the t-statistics associated with are somewhat lower because individual effects tend to offset each other.

While the statistics reported in Tables 1 and 2, as well as comparisons across models (there are no sign disagreements for significant parameters), give us confidence in the results, the implied magnitudes should be taken cautiously. The log-linear form of the models makes elasticities highly sensitive to their points of measurement. Furthermore, each elasticity changes sign if the variable it pertains to changes sign. We reverse such sign switches in Table 2, so that the signs of all reported elasticities reflect the direction of the relationship between the relevant variable and capacity.

The highly significant positive impact of reserves on capacity does not come as a surprise; despite the fact that reserve data are often considered unreliable as indicators of ultimate cumulative extraction, this confirms that proven reserves at the start-up date are crucial to the choice of the scale of operation. The elasticity of 0.95 at sample mean is reasonable; however this figure should not be extrapolated: the log-linear form implies a higher elasticity at higher reserve levels, while, in practice, the elasticity should vanish with the scarcity constraint. Perhaps more surprisingly, we did not identify any significant impact of as an indicator of technical advancement, on capacity (hence the omission of that variable in the tables). This does not imply the absence of any technological change; just the absence of scale augmenting technological change. In contrast, the type of exploitation method, as represented by (1 for open-pit mines; zero otherwise) has a significant impact in all three model categories.

The output price via and also has a significant impact on

\footnote{If the elasticity also depends on variables other than the variable under focus (e.g. (35) below depends on q and A, besides ), these are evaluated at their sample mean.}
capacity. At 3.09 (with tax holiday) and 2.87 (without tax holiday), the sample mean elasticity is rather high. Even though the elasticity magnitudes presented here should be taken cautiously, this is indicative of a stronger price effect than is usually found in standard neoclassical supply models. A possible explanation is that the ‘putty-clay’ model focuses on situations where the flexibility of firms is constrained by blueprints only, while neoclassical models assume, but do not find, the same kind of flexibility on the part of firms that are already constrained by existing plants. To present the same argument from a different perspective, the fact that our sample is restricted to firms that do invest, thus eliminating observations where capacity would remain unchanged at zero or at any positive level, implies a more sensitive concept of elasticity. Although this warrants further research, we believe that identifying a significant influence of output price where no other study of (non-energy) extractive resource supply has found any significant effect, is a direct consequence and confirmation of the ‘putty-clay’ assumption. Similar comments apply to the influence of variable factor prices, which is both strong and highly significant. Interpretation is facilitated by noting that mines are so capital-intensive that capacity may be identified with capital stock. Thus energy and materials appear to be complements of capital, while labour is a substitute.

Because \( WK = \delta Q A_0 \), one might expect \( \varepsilon_{Q, \delta} \) to equal \( \varepsilon_{Q, WK} \). In fact, the former is different, and has a different sign. This happens because, besides its influence on capacity via the cost of capital, \( \delta \) also enters the model independently, which adds an extra term in formula (35). As explained in the theoretical part, a rise in \( \delta \) reduces the opportunity cost of reserves, which mitigates the negative impact of a higher cost of capital and explains the difference.

3.4. The real impact of taxation

The coefficient of \( WK \) is significantly negative, confirming that the after-tax cost of capital affects capacity in the expected way. Since \( WK = \delta Q A_0 \), the elasticity of capacity with respect to \( A_0 \) is given by \( \varepsilon_{Q, WK} \). \( A_0 \) is positive in the first part of the sample, and becomes negative after the 1972–1976 reforms, so that its mean value is close to zero, and the corresponding elasticity of \(-0.35\) is probably an understatement; the values of \(-1.23\) and \(-1.46\) at the sample minimum and maximum are probably more representative of the post-reform and the pre-reform situations respectively. Effective tax rates also affect capacity, jointly with output price. However, while there is only one price, our model makes a distinction between effective tax

\[17\] On oil extraction, see, for example, Pesaran (1990).
rates during (1-A1), and after (1-A2), the tax holiday. This is why the
elasticities associated with A1 and A2 are different from the price elasticity
when there is a tax holiday. As a matter of fact, A2 appears to matter more,
both in terms of magnitude and statistical significance, which may imply that
the tax holiday is not an important factor in capacity choice.

In fact, the importance of the tax holiday, via P1, T1, or REF, is difficult
to ascertain. The coefficient of T1 is significantly different from zero in only
two of the reported equations, and the coefficient of P1 is significantly
different from zero only in (33) and (34); in all three cases, the signs are as
expected. In (33) and (34), it appears that the dummy variable REF (=1
when the after-tax cost of capital is negative, i.e. after the 1974–1976
reforms which suppressed the tax holiday; zero otherwise) has more
explanatory power than T1. If it accounted for the influence of the tax
holiday in isolation, the sign of its coefficient should be negative. One may
interpret the positive sign as reflecting the fact that the 1974–1976 reforms
included (favourable) compensatory changes in other tax parameters.

Inconclusive as to the effect of the tax holiday, our results confirm other
theoretical predictions: low marginal tax rates, and a low after-tax cost of
capital, induce a higher capacity. However, as in most similar studies, while
we have established that after-tax output and factor prices have a significant
impact on capacity decisions, we do not know whether the impact of taxes is
statistically significant, as distinct from the impact of pre-tax prices. It is
desirable to test whether taxes have a significant effect of their own. If they
do not, we could conclude that taxation is statistically neutral, as if A_0, A_1,
and A_2 were all equal to one.

In order to answer this question, we have to test whether our model
statistically differs from an alternative, non-nested, model featuring tax
neutrality. We apply the J-test of Davidson and MacKinnon (1981) to (28),
(32), and (34). Thus, in the case of (34), the competing hypotheses are

\[ H_0 : \ln Q = X\beta_0 + \mu_0 , \]

where \( X = (\text{Constant}, P1, P2, WK, WL, WE, WM, \delta, R, EXPL, REF) \); and

\[ H_1 : \ln Q = Z\beta_1 + \mu_1 , \]

where \( Z \) is a matrix of the same explanatory variables as \( X \), except that the
variables \( A_i (i = 0, 1, 2) \) are set equal to one in the computation of \( P1, P2, \)
and \( WK \).

The J-test consists of embedding the alternatives into an extended model
using a mixing parameter \( \lambda \):

\[ \ln Q = (1 - \lambda)X\beta_0 + \lambda Z\beta_1 + \mu , \]
where $\mu$ is an i.i.d. error term of mean zero. Replacing $Z\beta_1$ with its estimate $\ln Q_1^* = Z\beta_1^*$ under $H_1$:

$$\ln Q = (1 - \lambda)X\beta_0 + \lambda \ln Q_1^* + \mu.$$  

If $H_0$ is true, then $\lambda = 0$, which may be tested using the $t$-statistics. Note that an alternative procedure might consist in replacing $X\beta_0$ with its estimated value under $H_0$. Bernanke et al. (1988) discuss non-nested specification tests in a wider set of circumstances, raising the issue of possible intransitivities. Given that there are only two alternatives to be considered here, that possibility is remote. Nonetheless, we applied both alternative procedures to (28), (32), and (34); in all cases, $H_1$ is rejected in favour of $H_0$. For example, the standard procedure applied to (34) gives $\lambda = -0.29$ ($t = -0.15$).

The procedure just described represents a joint test of the importance of $A_{0_1}$, $A_1$, and $A_2$ for capacity decisions. However, it may be that firms are influenced by the taxation of net revenues but do not care about capital allowances ($A_0 = 1$), or the converse ($A_1 = A_2 = 1$). We also tested these more specific hypotheses by the same procedure, finding that the appropriate versions of $H_1$ could be rejected in favour of $H_0$ in both instances, with $\lambda = -0.34$ ($t = -0.19$) when $H_1$ is $A_0 = 1$ and $\lambda = 0.17$ ($t = 0.09$) when $H_1$ is $A_1 = A_2 = 1$.

4. Conclusion

In this paper we examined the effect of taxation on capacity choice by individual Canadian copper firms during the period 1960–1980. Our model is characterized by ‘putty clay’ technology. Tax provisions from three administrative regimes, differing according to date and location, were combined into four basic tax variables: one variable measuring the after-tax cost of capital; two variables reflecting effective tax rates, respectively during, and after, the tax holiday; the fourth variable giving the duration of the tax holiday, if any. A fifth (dummy) variable was also introduced in some versions of the model to pick up the global impact of an important series of reforms. Although the precise magnitudes remain debatable, empirical results confirmed the importance of taxation, together with prices.

18 More precisely, for this, and the other two tests reported below, one procedure leads to the non-rejection of $H_0$ in favour of $H_1$, while the alternative procedure leads to the rejection of $H_1$ in favour of $H_0$. 

and geological variables, as determinants of capacity. While generally considered important by both academic and industrial specialists, such a role had not been identified precisely in any earlier econometric study. Our modelling approach probably accounts for the fact that our results were more conclusive. Since several other industries exhibit characteristics similar to mines, many empirical studies of taxation should probably be carried out within 'putty clay' models, along the methodological lines presented in this paper, rather than under the prevailing full-malleability assumption. Another innovation was our attempt to separate out the role of taxation from that of pre-tax prices in statistical tests.

Many issues remain unaddressed in the area of investment and capacity decisions, however. Perhaps the most important one is the timing of such decisions. This paper has addressed the issue of capacity choice given that investment was occurring. How firms choose the date of their investment is another, difficult issue. When investment is irreversible and bulky, and must be made under uncertainty, it is well known that the positive net-present-value criterion does not apply. Waiting yields information, whose value must be incorporated into the decision process. For a few years, several authors (see Pindyck, 1991, and the references therein) have adapted the theoretical framework of option theory to this problem, thus treating an irreversible physical investment as the exercise of an option, with uncertainty arising from markets or taxes (Mackie-Mason, 1990). However, their simulations have been based on investment projects whose real magnitude was independent of economic conditions. In this paper, the physical size of the projects is sensitive to economic variables. In fact, it is possible to simulate from our results what capacity would have been selected by any given mine if it had chosen a different start-up date. This opens up an opportunity to test the validity of option theory as a theory of real investment, which we hope to exploit in another paper.

Acknowledgements

The suggestions made by Marcel Dagenais, Christian Gouriéroux, Claude Montmarquette, Pierre Perron, and Robert Pindyck are gratefully acknowledged. We thank two anonymous referees for suggesting substantial improvements, including the correction of an error in our treatment of the price process. Remaining errors are our own. Both authors thank the SSH RCC funds for financial assistance.

Appendix A: Proofs of properties

(i) $W^*$ can be shown to be convex exactly in the same way as a static profit function can be shown to be convex (Silberberg, 1990, p. 193).

(ii) Homogeneity is established by showing that the maximization of $W$ gives the same result, up to a multiplicative scalar $\lambda$, if the three prices are multiplied by that scalar, or if the three tax parameters are multiplied by that scalar.

(iii) From (ii) it follows that $W^*$ is homogeneous of degree two in $(p, w, q, A_0, A_1, A_2)$. Consequently, by the Euler theorem, $\Sigma^*$ is homogeneous of degree one in $(p, w, q, A_0, A_1, A_2)$. Now consider (21); multiply $(p, w, q, A_0, A_1, A_2)$ by $\lambda$; then, to satisfy the homogeneity of $\Sigma^*, Q^*$ must remain unchanged. Consequently, $Q^*$ is homogeneous of degree zero in $(p, w, q, A_0, A_1, A_2)$. Also, from (ii), by the Euler theorem, $\Sigma^*$ is homogeneous of degree zero in $(p, w, q)$. Considering the homogeneity of degree one of $\Sigma^*$ in $(p, w, q, A_0, A_1, A_2)$ and (21), this implies that $Q^*$ is homogeneous of degree zero in $(A_0, A_1, A_2)$; it follows that $Q^*$ is also homogeneous of degree zero in $(p, w, q)$.

(iv) From (i), $\partial K^*/\partial q \leq 0$ and $\partial \Sigma^*/\partial p \geq 0$. Differentiating (21) with respect to $p$, using $T^* = R / Q^*$, we get

$$
\frac{\partial \Sigma^*}{\partial p} = \frac{\partial Q^*}{\partial p} \frac{1}{\delta} \left[ 1 - (\delta T^* + 1) e^{-\delta T^*} \right]. 
$$

(A.1)

If $\delta$ is positive, then the expression between square brackets in (A.1) is positive. This can be shown by noting that, for $T^* = 0$, this expression is equal to zero; since its derivative with respect to $T^*$ is positive, it follows that the expression is positive for any positive $T^*$, so that the sign of $\partial Q^*/\partial p$ is the same as the sign of $\partial \Sigma^*/\partial p$.

The proofs that $\partial Q^*/\partial A_1$ and $\partial Q^*/\partial A_2$ are non-negative are similar, but start from a different convexity property: by visual inspection of (13) and (20), one notes that $W$ and $W^*$ can be redefined as functions of $pA_1, pA_2, wA_1, wA_2, qA_0, etc.$ instead of $p, w, q, A_0, A_1, A_2, etc.$ Then, noting $W'$ the function corresponding to $W^*$, as redefined in terms of the new variable set, it can be shown, as with $W^*$, that $W'$ is convex in $pA_1, pA_2, wA_1, wA_2, qA_0$ and that the supply functions analogs to (21) are

$$
\frac{\partial W'}{\partial (pA_1)} = \frac{(1 - e^{-\delta T^*})}{\delta} Q^* = \Sigma_1.
$$

(A.2)

and

20 Clearly if the original six variables can be treated as independent, so can the new subset of five variables.
\[
\frac{\partial W'}{\partial (pA_2)} = \left( e^{-\delta T} - e^{-\delta T'} \right) Q^* = \Sigma_2 \tag{A.3}
\]

Differentiating (A.3) with respect to \( A_2 \), holding \( p \) constant, we have

\[
\frac{\partial^2 W'}{\partial (pA_2) \partial A_2} = \frac{\partial^2 W'}{\partial (pA_2)^2} = \frac{(e^{-\delta T} - e^{-\delta T'})}{\delta} \frac{\partial Q^*}{\partial A_2} + Q^* e^{\delta T'} \frac{\partial T^*}{\partial A_2}.
\]

Using \( T^* = R/Q^* \), and rearranging, we obtain:

\[
p \frac{\partial^2 W'}{\partial (pA_2)^2} = \left[ \frac{e^{-\delta T} - (1 + \delta T^*) e^{-\delta T'}}{\delta} \right] \frac{\partial Q^*}{\partial A_2}.
\]

Let \( T' \) be defined by the condition that the term between square brackets be null. Since its derivative with respect to \( T^* \) is positive, that term is non-negative for any \( T^* > T' \), and negative otherwise. It follows that, for any \( T^* \geq T' \) (\( T^* < T' \)), \( \partial Q^*/\partial A_2 \) has the same (opposite) sign as the left-hand side, which is non-negative by the convexity of \( W' \). That \( T' > T_1 \) follows from the observation that the term between square brackets is negative when \( T^* = T_1 \). The proof that \( \partial Q^*/\partial A_1 \) is non-negative is similar, but based on (A.2).

The proof that \( \partial Q^*/\partial T_1 \geq 0 \) if \( A_1 \geq A_2 \) is again similar. This time we redefine \( W^* \) as a function \( W^* \) of \( PT, WT, pA_2, wA_2, qA_0, \) etc. where

\[
PT = \frac{1}{\delta} \left[ A_1 - e^{-\delta T_1} (A_1 - A_2) \right] p
\]

and

\[
WT = \frac{1}{\rho} \left[ A_1 - e^{-r T_1} (A_1 - A_2) \right] w.
\]

Hotelling’s lemma gives \( \partial W'/\partial (PT) = Q^* \) and \( W' \) is convex. Differentiating \( Q^* \) with respect to \( T_1 \) holding \( p, A_1, \) and \( A_2 \) constant

\[
\frac{\partial Q^*}{\partial (T_1)} = e^{\delta T_1} (A_1 - A_2) p \frac{\partial^2 W'}{\partial (PT)^2},
\]

from which the desired result follows immediately.

That \( \partial Q^*/\partial w, \partial Q^*/\partial \delta, \partial Q^*/\partial p \) may be positive or negative may be proven using examples. This is analogous to the property that the partial derivatives of static supply curves with respect to factor prices cannot be signed in general.

The proof that \( \partial Q^*/\partial R \leq 0 \) if \( K^* \) does not decrease as \( Q^* \) increases is lengthy. It will be made available to any interested reader upon request.
Appendix B: The data

This appendix lists the major capacity investments used as observations, and describes the variables. Sources are displayed in roman numbers between parentheses and listed at the end of the appendix.

Observations
(2) Vauze Mines, Quebec, 1961.
(3) Bethlehem Copper, British Columbia, 1962.
(10) Bethlehem Copper, British Columbia, 1966.
(13) Prace Mining Corp., Ontario, 1967.
(20) Lornex Mining Corp, British Columbia, 1974.
(22) Lornex Mining Corp, British Columbia, 1979.

Variables
$A_n =$ after-tax cost of a $1 capital expenditure, ignoring the effect of inflation (III and V).
$A_i =$ after-tax revenue per dollar of gross income, during the tax holiday (set to zero in the absence of tax holiday) (III and V).
$A_3 =$ after-tax revenue per dollar of gross income after the tax holiday period (III and V).
$i =$ long-term rate of inflation in Canada measured as a five-year moving average of the rate of growth of the consumer price index, $100 = 1971$ (VIc and VIa).
\( p \) = price of one metric ton of copper in nominal Canadian dollars, 100 = 1971, based on the New York metal exchange (IV and VIa).

\( q \) = implicit nominal price index of machinery and equipment, 100 = 1971 (I and VIc).

\( Q \) = capacity of the mining firm in metric tons of ore per day (II).

\( r \) = real discount rate, defined as the nominal 10 year Canada bond rate (VIa), minus \( i \).

\( R \) = proven reserves in 10^6 metric tons of ore (II).

\( WE \) = Törnqvist nominal price index of energy in the mining industry, 100 = 1971 (VIb and VIId).

\( WL \) = nominal wage rate index in the mining industry, 100 = 1971 (VIId).

\( WM \) = (as a proxy for the nominal price of materials) sales price index for manufacturing industries, 100 = 1971 (VIc and I).

\( s \) = start-up date for the new plant (see list above) (II).

\( T1 \) = duration of the tax holiday (III and V).

\( \alpha_p \) = average yearly growth rate of real copper prices in Canadian dollars over the period 1940–s (IV and VIa).

\( \delta \) = \( r - \alpha_p \) = adjusted discount rate for revenues.

\( \rho \) = \( r - \alpha_w \) = adjusted discount rate for expenditures (see text).

Sources

(1) Bank of Canada Review, quarterly, different issues, Ottawa.


(IV) Commodity Trade and Price Trends, different issues, World Bank, Washington, DC.


(VIb) Statistique Canada, Consommation de combustibles et d’électricité achetés par les industries manufacturières et minières et par les centrales thermiques des services d’électricité, hors série, 57–506, Ottawa.


(VId) Statistique Canada, Revue générale sur les industries minières, 26–201, Ottawa.
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